

FORMAL ONTOLOGY AND DYNAMIC ONENESS

EXPLORING THE ONTOLOGICAL FOUNDATIONS OF LOGIC

di
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The current of thought that flows from the spiritual heritage of Chiara Lubich has begun to have impact in a number of different fields of investigation and scholarly pursuit. This brief article presents a reflection on the theme of dynamic oneness from the viewpoint of logic, or more specifically of "formal ontology."
For Edmund Husserl formal ontology, closely related to formal mathematics, expands the scope of logic to include not only forms of judgements but the complementary realm of objects, considered in their underlying structural makeup.
The axiomatic framework elaborated by the mathematician Ennio De Giorgi can be taken as a point of departure for a regressive inquiry towards a more primordial, elementary category of relationality suitable for expressing the specific pattern of dynamic oneness under study in terms of formal ontology. The recent attempt of L. Obojska in this direction introduces new formal notions and terms and provides ground for future in-depth research and interdisciplinary exchange.

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"We are at a moment of history particularly suited for carrying out a reflection on logic, not considered alone, in an isolated way, but in connection with all other spheres of knowledge…."[1]
The current of thought that flows from the spiritual heritage of Chiara Lubich has begun to have impact in a number of different fields of investigation and scholarly pursuit. One can note that the underlying theme, although expressed in the particular categories and specialized terminology of these diverse ambits, remains essentially unaltered. It is a leitmotif that regards a form of interrelatedness that has yet to be studied adequately in its ontological roots, a pattern of dynamic oneness that presupposes a total, reciprocal self-giving of distinct agents and leads to a mutual immanence that is reminiscent, for theologians, of the doctrine of the Trinity.
This brief article presents a reflection on this theme from the viewpoint of logic, not in the traditional sense of logic as devoted to forms of correct reasoning disjoint from objects, but more in line with the complimentary area indicated by Edmund Husserl as "formal ontology."[2] In so few pages, we can obviously make no attempt to be scientifically adequate but would like merely to indicate areas for further exploration and the possibility of new paths to trod.
After a premise presenting a few points of Husserl's concept of formal ontology, we will illustrate an axiomatic framework elaborated by Ennio De Giorgi as a possible point of departure for the task we have set out to achieve: to express the pattern of interrelatedness indicated above in terms of formal ontology. We will conclude by describing the recent attempt of a young Polish mathematician, Lidia Obojska, together with a group of colleagues, to bring forward this proposed endeavour.

Formal Ontology

To begin, let us say a few words regarding formal ontology. As it is well known, in the science of formal logic it is the form that matters, i.e., the relation between words and phrases, rather than the individual words or phrases themselves. It was Aristotle's merit to have discovered this, as he put some order into the confusion of ideas that reigned in his time by drawing a distinction between the ontological level and that of linguistic or logical correctness.[3] Once the tenets of formal logic had been laid down, it became clear that proper reasoning was governed

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1) G. Bigotti, *Logica*, in *Dizionario Interdisciplinare di Scienza e Fede*, ed. G. Tanzella-Nitti and A. Strumia, Urbaniana University Press/Città Nuova, Rome 2002, p.805 (our translation).
2) E. Husserl, *Formal and Transcendental Logic*, tr. D. Cairns, Martinus Nijhoff, The Hague 1968, pp. 71-86.
3) The pioneers of logic in ancient Greece were concerned with laying down rules for clearness and correctness of speech and thought. There was a need to understand how altering forms of reasoning could lead in subtle ways to opposing philosophical tenets, such as—on the one hand—that "everything flows and nothing stays," as Heraclitus (c. 535–c. 475 B.C.) had stated, and—on the other—that motion is altogether impossible, as Zeno, disciple of Parmenides, had set out to prove. Indeed, in reply to the Sophists, who used language and reasoning to reach paradoxical results, it seemed necessary to defend the very possibility of knowing anything at all!

by general rules of relatedness or form, whichever language one would choose. In fact, Aristotle introduced the use of symbols (letters) in the place of objects or predicates in his descriptions of syllogisms, evidencing in this way that the object of investigation in this case was the form, not the meaning.[4]

Modern mathematical logic grew naturally out of this same realization; for mathematics, too, is a kind of language. Indeed, from Boole to Frege to the contemporary burgeoning of computer science, mathematics has proved to be a useful and particularly suited language for expressing in abstract forms the varieties and modes embodied by logical thinking. It is upon this edifice, moreover, that modern science stands.

However, as Edmund Husserl, prophet of the crisis of science, pointed out, there is a need for a new logic to fill in the formal structures of traditional logic, to "let fullness flow back into these cores, which have been kept emptily universal."[6] Just as it has become clear in the realm of scientific thought that determinism must be counterbalanced by alternate perspectives, traditional logic and the systems that incorporate it also need to be considered as being a part of a wider picture.

What did the renowned German philosopher have in mind? Needless to say, the method and objectives of Husserl's system of phenomenology, as well as the insights and problems raised by it, are beyond the scope of this paper. Without attempting more than a glimpse at his deep and at times ambivalent system of thought, we would like to highlight a few passages regarding that area of investigation that Husserl termed "formal ontology," through which he expanded the scope of logic to include not only forms of judgements but the complementary realm of objects, considered in their underlying structural makeup.

"Formal ontology is the theory of logic considered from the point of view of the object of any possible judgement, dislocated from any particular one; it deals with the most general categories of the object (objects, sets, numbers, facts, equality, identity, relation, and so on)."[7]

One can note in the categories of objects mentioned here an affinity with the realm of formal mathematics. Indeed, Husserl explicitly confirms mathematics as being the inspiring model upon which he based his ideas. It is said that mathematics was Aristotle's source of inspiration regarding the rules for reasoning, which he crystallized into principles of logic; for Husserl mathematics provided insight regarding the forms of objects.

Note

4) *Prior Analytics*, 2ss.

5) E. Husserl, *Formal and Transcendental Logic*, 150. Cf. also *The Crisis of European Science and Transcendental Phenomenology*, tr. D. Carr, Northwestern University Press, Evanston 1970.

6) For a competent exposition of the topic, cf. B. Smith, *Logic and Formal Ontology*, in J.N. Mohanty and W. McKenna, eds., *Husserl's Phenomenology: A Textbook*, University Press of America, Lanham 1989, pp. 29–67. For an analysis of formal ontology in connection with the object of science, cf. P. Komesaroff, *Objectivity, Science and Society*, Routledge & Kegan Paul, London 1986, pp. 103–170.

7) Komesaroff, *op. cit.*, p. 115; cf. Husserl, *Formal and Transcendental Logic*, pp. 24–25.

The theory of sets and the theory of cardinal numbers relate to the empty universe, any object whatever or anything whatever, with a formal universality that, on principle, leaves out of consideration every material determination of objects.… It is natural to view this whole mathematics as an ontology (an a priori theory of objects), though a formal one, relating to the pure modes of anything—whatever. In so doing, one would also acquire the guiding idea for determining the separate provinces of this ontology—this mathematics of all objectivities as such—by a priori structural considerations.[8]

For Husserl then, formal ontology regarding the most general forms of objects and formal mathematics were closely interrelated, at times indistinguishable. In light of these considerations, it would certainly seem appropriate that mathematicians and/or mathematical logicians try to work out a formal elaboration of the general relational pattern described at the outset of this paper. The results of such an effort could then serve as a conceptual basis for other disciplines and a general foundation for clarity of thought and the development of new insights.

De Giorgi's Axiomatic System

Ennio De Giorgi, reknowned Italian mathematician, winner of the coveted Wolf Award in 1996 for his work on differential equations and complex manifolds, would surely have subscribed to this view. De Giorgi was neither a philosopher nor a logician, but as a humanist, beyond his specific profession, he was keenly interested in transdisciplinary dialogue.

In De Giorgi's writings, we note his deep awareness of the fragmentation of knowledge that plagues learning today and the all-too-frequent incommunicability between specialized fields of intellectual inquiry, each with their own terminology and specific background knowledge. He sensed, moreover, the need to bring together mathematical formalists and Platonists in complementary collaboration, certain that new intellectual paths are often forged through a capacity or readiness to "dream," although, at the same time, innovative intuitions must be rendered clear and communicable through rigorous formulation.

On the one hand, throughout his professional life De Giorgi had come to appreciate the universal nature of the language and culture of mathematics, which permits ambiguity-free dialogue and sincere respect between scholars of distant nations and cultures. On the other, he was convinced of the importance of interdisciplinary communication on a vast scale, noting that in scholarly progress, "the most interesting ideas have often 'come from afar,' suggested by analogy, considering facts and theories from branches of knowledge that are very distant from one's own discipline…."[9]

Thus De Giorgi's project and vision was to create a basic axiomatic framework that

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8) Husserl, *Formal and Transcendental Logic*, cit., pp. 77–78.

9) F. Bassani - A. Marino - C. Sbordone, eds., *Ennio De Giorgi. Anche la scienza ha bisogno di sognare*, Università di Pisa, Pisa 2001, p. 193 (our translation).

would help facilitate cross-disciplinary exposure and understanding by going back to pre-logical, pre-mathematical roots.

This attempt was to serve a twofold purpose: first, to formulate the general discourse on mathematics, logic, and informatics in a clear and comprehensible way, so that the significant intellectual insights gained in these fields might be "accessible to all those scholars of scientific and humanistic disciplines who are animated by what the ancients called philosophy, i.e., love of Wisdom."[10] Second, by nesting mathematical theories, and hopefully other scientific and non-scientific theories, into a broad underlying framework, it was hoped that it would be possible to evaluate and compare the basic ideas of the various disciplines in a clear, more objective way.

In a sense, De Giorgi's project might be considered as fitting into a more vast contemporary movement of thought aimed at creating a scientific interlingua, or, as some have termed it, a scientific lingua franca.[11] De Giorgi's scope, however, was more universal, and his method distinctive. It was not so much a question of creating a metalanguage for science but rather identifying a small number of the most general concepts underlying all human discourse, expressing these formally and axiomatically as, being a mathematician, he had been trained to do. We have no indication that De Giorgi was aware of Husserl's proposal of phenomenology, but certainly his ideas would seem to fit in with Husserl's call for a formal ontology.

De Giorgi worked on this pet project together with Marco Forti and Giacomo Lenzi at the Normal School of Pisa for several years before his untimely death in 1996. The initial results were published in several Preprints of the Normal School of Pisa and later continued by Forti and Lenzi.[12] The authors outlined what they saw as a simple, clear and "natural" axiomatic basis upon which to graft—to begin with—the various branches of mathematics, logic, and informatics, then subsequently "some fundamental ideas of other scientific and humanistic disciplines, opening new vistas onto thought concerning the relations between various fields of study."[13]

It is interesting to note that the pivotal, undefined concepts in De Giorgi's axiomatic system are "quality" and "relation," identified as fundamental underlying elements of any sector of knowledge. These "ur-elements" are expressed in a simple symbolic language reminiscent of formal logic or mathematics, but not reducible to these. In fact, inversely, it is after the introduction of the basic axiomatic structure that fundamental mathematical and logical concepts are subsequently defined by the author in terms of quality and relation.

Note

10) E. De Giorgi - M. Forti - G. Lenzi, *Verso i sistemi assiomatici del 2000 in matematica, logica e informatica*, *Scuola Normale Superiore di Pisa, Preprints di Matematica* 26 (1996): 1–19 (hereafter *Verso i sistemi assiomatici*) (our translation).

11) Cf., for example, U. Eco, *The Search for the Perfect Language*; Blackwell 1994.

12) Cf. E. De Giorgi - M. Forti - G. Lenzi, *op. cit.*
Cf. also E. De Giorgi - M. Forti - G. Lenzi, *Verità e giudizi in una nuova prospettiva assiomatica*, *Scuola Normale Superiore di Pisa, Preprints di Matematica* 32 (1996); M. Forti, G. Lenzi, *A General Axiomatic Framework for the Foundations of Mathematics, Logic and Computer Science*, in *Rend. Mat. Acc. Naz. Sc.* d. XL (1997): 1–32.

13) E. De Giorgi - M. Forti - G. Lenzi, *Verso i sistemi assiomatici*, cit., p. 2.

Perhaps, in summing up De Giorgi's program, we can point out that while quality and relation remain primitive notions, through descriptive axioms they are characterized as being mutually dependent. In fact, the axiom describing "quality" makes reference to a specific binary relation that joins a quality to the entity possessing that quality. On the other hand, the axiom describing a general binary relation (relb) begins by noting that any binary relation has first of all the quality of being a relation (Q relb). Thus the terms "quality" and "binary relation" are mutually referential, whereas the term "entity" remains the undefined substrate of the entire discourse.

Beyond De Giorgi

Like Leibniz, Frege, and others before him, De Giorgi had seen the need for a sort of universal language and directed his mathematical talents toward a project bent on realizing this dream in a unique way.
There have already been two promising initial efforts to incorporate this axiomatic language or framework, in the sense of De Giorgi's vision, in specific areas of research. In the field of biology, various articles of Lodovico Galleni, colleague of De Giorgi at Pisa, translate key concepts of the life sciences into the proposed axiomatic language.[14] In the field of logic and formal ontology, we have the recently published article of Lidia Obojska.[15] The latter is the fruit of the collaborative research of a group of mathematicians of various specializations aiming to express in unambiguous, formal language the pattern of dynamic oneness perceived in the ontological foundation of the charism of unity of Chiara Lubich.[16]
In Obojska, the ur-concept of binary relation, as singled out and described axiomatically by De Giorgi, becomes the point of departure for a regressive inquiry in the direction of a more primordial, elementary category of relationality. Because

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14) Cf. L. Galleni, M. Forti, *An Axiomatization of Biological Concepts within the Foundational Theory of Ennio De Giorgi*, in *Rivista di Biologia/Biology Forum* 92 (1999): 78–104; L. Galleni, *Foundations of Biology and the Search for an Axiomatic Language*, in *Aquinas*, XLIII (2000): 369–379. Thanks to a common framework of understanding, we were able to interact with the author regarding some key points of his articles. Particularly significant was his following reply: "I was pleased to see that although you are not a biologist, you have perfectly centred the heart of the problem….This shows that our aim in using De Giorgi's language has been achieved: those who are not experts in a discipline but know this language can enter into the problems and discuss them…De Giorgi's framework is also very useful in that with it we do not presume to completely comprehend a given reality, but to describe it in a manner in which the description can be discussed and compared with others." (our translation).
15) L. Obojska, *Primary Relations in a New Foundational Axiomatic Framework*, in *Journal of Philosophical Logic* 36 (2007): 641–657 (hereafter *op. cit.*). See also L. Obojska, *Primary Relations and a Non-Standard Form of Identity*, in P. Valore ed., *Topics on General and Formal Ontology*, Polimetrica Publisher.
16) Cf. the parallel efforts of the interdisciplinary group coordinated by the Philosophical Institute of Thomastic Studies of Modena to express Aquinas's concept of analogy in the language of formal logic in *Analogia e autoreferenza*, ed. G. Basti e C. A. Testi, Marietti, Genoa 2004, particularly the chapter by C. Testi, *Analogia, logica formale e paradossi*.

the categories presented by De Giorgi proved inadequate for describing the pattern of interrelatedness presented at the start of our essay, Obojska introduced the concept of "primary relation" and, more importantly, a specific form of primary relation that she defines as "tr relation."

In reality, entities termed unary relations are not a novelty on the mathematics scene.[17] In Obojska's case, however, the concept of primary relation (corresponding to the mathematical concept of unary relation)[18] is more universal, flowing naturally from De Giorgi's scheme, which is pre-mathematical in nature. Thus it is an invitation to reflect on the inherent meaning of unary relation, while at the same time opening toward more general, non-mathematical fields of relevance.

In De Giorgi's formal system, for any given relation (binary, ternary, or quaternary) there is a corresponding fundamental relation, which describes the way in which the given relation is joined to the entities related. He notes, for example, that for a binary relation, in considering the system of "binary relation plus 2 entities related," we have a system of three entities, and therefore the corresponding fundamental relation is ternary. Similarly, the fundamental relation that joins a ternary relation to the three entities related is quaternary, and so forth. Generalizing, the fundamental relation is always of one order higher than the relation considered.

In continuity with De Giorgi's scheme, Obojska describes a primary relation by means of a fundamental relation that is binary. Intuitively speaking, based on De Giorgi's description of binary relation, an external observer would be able to note a system with three entities (two being joined, plus a binary relation joining them). Analogously, according to Obojska's description of primary relation, an observer looking from above or outside would note a system with only two entities, the primary relation being itself one of the related objects. This is a rather unrefined, intuitive portrayal of what Obojska defines formally and elegantly in a single line, grafting it onto the terminology introduced by De Giorgi.

Of course, the general concept of primary relation cannot be intuitively understood immediately; we are not used to reasoning in these terms. Perhaps it would be helpful to give some illustrative examples: the relation of a part to a whole is aptly expressed in terms of primary relation, for the part is in the whole, and not conceivable without it. Thus the part itself, with respect to the whole, represents a primary relation. In fact, the two terms are coessential (mutually dependent), and it is a misconception to consider them as existing separately and distinctly at the same time. Similarly, a subset can be defined as a primary relation with respect to the set that contains it, and a topological boundary as a primary relation with respect to that which is bounded. In less mathematical terms, we suggest as primary relations the relation of a living creature to the environment in which it lives, that of a cell to an organism, etc.

Note

17) In the field of algebra, H. Rasiowa considers properties, and consequently subsets, in terms of unary relations. However the author does not develop the concept of unary relations beyond this. (H. Rasiowa, *Wstęp do matematyki współczesnej (Introduction to modern mathematics),* Wydawnictwo Naukowe PWN, Warsaw 1968.

18) One could note in the terminology an unintentional but fortuitous parallel with Aristotle's "primary substance."

As an immediate result of what has been said and of the De Giorgian axioms defining quality and the classical identity relation, both of these concepts can now be described in terms of primary relations. Moreover, if quality is a form of relation, the axiomatic framework of De Giorgi could, in effect, be reduced to a single primitive notion, that of relation.

Continuing on from here and using the newly defined notion of primary relation, Obojska goes one critical step further. Building on the general definition outlined above, she introduces a new kind of primary relation called "tr relation" (or "type-tr primary relation"), suitable for describing the pattern of interrelatedness evidenced at the start of this essay, which seems to defy the two cornerstones of classical Aristotelian logic: the principles of non-contradiction and the excluded middle. The technical apparatus, while simple in its formal representation, is beyond the scope of this paper.[19]

For the courageous reader, we can offer the following non-formal description: a tr relation is a relation whose corresponding fundamental relation is binary (thus, a tr relation is a form of primary relation) and bijective (one-to-one), for which the entities related by the fundamental binary relation are not the same object (tr is not the identity relation). It becomes evident that a tr relation cannot be time or space dependent.

It can be shown that a single given tr relation cannot be conceived without another corresponding non-identical tr relation. It is as if a tr relation, in its outpouring, generates a mirror image of itself. To distinguish between the two, Obojska uses the symbols tr_1 and tr_2. To describe tr_1 and tr_2 formally, we need to introduce concepts such as unary operations and mereological parts, which are again beyond the scope of this presentation. An interested reader can consult the scientific article cited previously.[20] Intuitively speaking, tr_1 is "in" tr_2, and tr_2 is "in" tr_1; they are distinct relations, yet there is nothing that can differentiate them. They are coessential and non-coexistent.

But, as the saying goes, where there are two, there are three. The formal determination of tr_3, through a description of its corresponding fundamental relation, is found to be intrinsically related to both tr_1 and tr_2, in a sense that is reminiscent of the mathematical relation "between." There is a difference, however, in that the relation "between," in its primitive logical sense, is a ternary relation joining three fixed objects, whereas tr_1, tr_2, and tr_3 are all dynamic tr relations. Intuitively, tr_3 is the dynamic component uniting and differentiating tr_1 and tr_2.

All three—tr_1, tr_2, and tr_3—are coessential, intrinsically and dynamically interrelated to form a oneness, a single whole that cannot be decomposed. It is a oneness-threeness model that Obojska calls the dynamic identity triple (DIT), or non-standard identity.

Note

19) For a formal definition of tr relation, see L. Obojska, *op. cit.*, Axiom 4.5.

20) L. Obojska, *op. cit.* The system of mereology founded by the Polish logician Lesniewski, upon which Obojska relies for a logical foundation of the axioms defining tr1 and tr2, and which is a crucial element in her elaboration, is based on a unique definition of part-whole relations. In addition, Husserl had worked out a detailed analysis of the relation of part to whole (Cf. E. Husserl, *Ricerche logiche*, tr. G. Piana, Net-Saggiatore, Milan 2005, V. 2).

Results, Limits, and Prospects

It would seem that Obojska, using the categories of De Giorgi and building on these, has successfully reached the goal of translating the pattern of interrelatedness indicated at the outset of this paper into terms of formal ontology. Beginning with an effort to describe this pattern of experience in ultimate substrates and relations, the author introduced new formal notions and terms—e.g., primary relation, tr relation, and dynamic identity triple—which are promising instruments for further research and analysis.

One might complain, so to say, that the medicine is worse than the malady. In our opinion, the main shortcoming of Obojska's work is the rather complex formulation of tr relations that involve notions of mereology, which is not immediately comprehensible. Thus a non-expert, without exposure to the object of experience she is attempting to describe, would find it difficult to grasp and use these results directly. On the other hand, the axiomatic framework is straightforward and coherent, and new ontological vistas are opened. Once an insight of the underlying pattern has been gained, the new concepts formulated can be useful vehicles of thought, offering insights in various fields.

To quote Husserl, "Logic relates not to what is given only in active evidence, but to abiding formations that have been primally instituted in active evidence…; it relates to them as objectivities which are henceforth at hand, with which…one can operate in thinking, and which…one can further shape categorically into more and more new formations."[21]

Perhaps a way to achieve greater clarity for Obojska's scheme lies, as in the quote at the start of our article, with a proposal of "logic, considered not alone, in an isolated way, but in connection with all the other spheres of knowledge…" Additionally, at this point, De Giorgi's suggestion that "there are probably many analogies that still need to be examined between the study of the fundamentals of mathematics and that of the fundamentals of metaphysics and of natural theology"[22] seems to have special bearing.

I am convinced that a form of nonclassical logic flowing from De Giorgi's axiomatic language and the added results of Obojska, with its new relational terminology (if further developed), could provide fertile ground for in-depth research in various fields and interdisciplinary exchange and communication.

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Note

21) E. Husserl, *Ricerche logiche*, tr. G. Piana (Milan: Net-Saggiatore, 2005), 185.

22) F. Bassani, A. Marino, C. Sbordone, eds., *op. cit.*, p. 179. He continues, "The reflection on these analogies could help in the elaboration of a metaphysics and of a theology which are mindful of the weaknesses of human reason but which do not forget their power and greatness, and we know that paradoxes can be overcome best by means of positive theories." De Giorgi was well aware then, as he says elsewhere, that "alongside the category of science characterized by very standardized forms of reason…there is the category of wisdom which is harder to grasp and define," of which mathematics is a branch, as are the other sciences (Ibid., 130).