Unveiling Paradoxes: Driving Progress in Physics

di Marco Sanchioni

This article delves into the evolving relationship between theoretical discoveries and empirical verification in physics. It addresses the challenges posed by the empirical incoherence of Quantum Gravity and explores the role of paradoxes as a guide in scientific progress. While experiments remain crucial for validating theories, the paper argues that paradoxes can provide valuable insights and motivate the development of new theories. By examining the concept of paradoxes, analysing their role in fundamental physics, and focusing on black hole paradoxes in Quantum Gravity, this paper proposes a heuristic approach to incorporating paradoxes into scientific inquiry. It emphasises the convergence of paradox analysis and experimental validation for a comprehensive understanding of the natural world.

Keywords: Paradoxes, Empirical verification, Quantum Gravity, Theoretical Discoveries, Scientific Progress, Black Hole Paradoxes

1. Introduction

In the vast domain of physics, the intricate relationship between theoretical discoveries and empirical verification has experienced a profound metamorphosis. In times gone by, our ability to test and validate theoretical hypotheses was hampered by a lack of technological tools. The immediate experimental confirmation of our theoretical predictions remained elusive due to these limitations. One of the most famous examples of a theoretical prediction we could not immediately test was the *Higgs Boson*, one of the last theorized particles of the standard model of particle physics, whose theoretical prediction was empirically corroborated at the Large Hadron Collider (LHC) only after approximately 40 years of experimental search.

Moreover, the contemporary scientific landscape presents us with an even more nuanced and complex situation regarding the interplay between theory and experiment. One of the foremost challenges arises from the empirical incoherence encountered in the realm of Quantum Gravity, i.e. the hypothetical theory capable of merging our two best scientific theories: General Relativity, and Quantum Mechanics. Indeed, it seems that, within Quantum Gravity, it is not only the case that we cannot test our physical theory in practice, i.e., we do not yet have the technologies to test them, but instead, we cannot test them *in principle*. There is a vast amount of literature on the fact that the fundamental research in theoretical physics concerning Quantum Gravity could be empirically incoherent because of the absence of spacetime in the fundamental ontology of the world. Without spacetime, we cannot do experiments, not even *in principle*, because experiments are necessarily located in space and time.[1]

This perplexing phenomenon continues to confound researchers, raising doubts about the viability of establishing a cohesive and comprehensive theoretical framework. However, even if we temporarily set aside the conundrum of the empirical incoherence of Quantum Gravity, a fundamental question emerges: can we continue to engage in scientific inquiry without substantial empirical evidence? Would such a departure from the traditional scientific method render our endeavours futile? Should we suspend our scientific pursuits until we can establish direct contact with experimental control?

In grappling with these weighty questions, I propose an alternative perspective. While empirical evidence undeniably holds immense value, I contend that

1 - See N. Huggett and C. Wüthrich, *Emergent spacetime and empirical (in)coherence,* in «Studies in History and Philosophy of Science, Part B: Studies in History and Philosophy of Modern Physics» 44 (3), 2013, pp. 276-285 for more on this problem. See M. Sanchioni, *Controllo in Fisica: sull'incoerenza Empirica della Gravità Quantistica*, Nuovo Sefir for an accessible discussion on this problem.

we can still make strides in scientific exploration by embracing a new form of guidance. Paradoxes, I argue, can serve as beacons illuminating our path forward. These paradoxes manifest as conflicts within our most established and trusted theories, highlighting the boundaries of our understanding. Instead of viewing them as obstacles, we should consider them opportunities for profound insights. Analysing and delving into the depths of these paradoxes can provide us with invaluable guidance in our research pursuits. They signify the boundaries of our current knowledge, showcasing the limitations of our existing theoretical frameworks. We embark on a journey of intellectual discovery by dissecting and exploring possible resolutions to these paradoxes. Such an approach can chart a course for future investigations, rendering experimental validation less immediate but no less significant.

Let me clarify that experiments remain a cornerstone of scientific progress. They serve as critical benchmarks for evaluating and refining our theories. However, in instances where experiments are lacking or unfeasible, paradoxes can be a prominent stepping stone towards progress. They function as a provisional framework, offering a starting point for theorising and exploring novel ideas until empirical evidence becomes available. Paradox analysis can serve as an initial step, igniting our scientific imagination and guiding our inquiries without direct experimental verification. It is important to note that the convergence of paradox analysis and experimental validation is vital for a robust scientific approach. The ultimate aim is to reconcile theory and experiment, creating a comprehensive understanding of the natural world.

On a different note, my paper can be understood as advocating for the attention that many physicists working on the foundations of Quantum Gravity have been dedicating to black hole paradoxes. I contend that these paradoxes can serve as a means to gain profound insights into Quantum Gravity's nature, even without immediate empirical data. By exploring and studying black hole paradoxes, researchers can delve into the depths of these enigmatic entities and unravel the underlying principles and structure of Quantum Gravity. It is crucial to acknowledge that black hole paradoxes do not replace the need for experiments. Empirical confirmation remains an essential step in the scientific process, as it allows for tangible progress in Quantum Gravity. However, black hole paradoxes can be pivotal in guiding researchers towards a more fundamental theory, even without direct empirical guidance.

In my discussion, I will begin by examining the concept of *paradox* as it pertains to physical phenomena. While a paradox is commonly understood as a counter-intuitive statement or situation that defies common sense, a more nuanced definition

is necessary. The philosophical literature offers various classifications of paradoxes, and Section

§2, I will propose several categories that capture the essential features of paradoxes in physics. Next, I will delve into paradoxes' role in recent research within fundamental physics, particularly when theoretical frameworks have surpassed our empirical capabilities. In Section §3, I will present three illustrative examples of paradoxes drawn from the annals of modern physics: the *train paradox* (§3.1), the *elevator paradox* (§3.2), and the *EPR paradox* (§3.3). In Section §4, I will argue that these three paradoxes have contributed to scientific progress in distinct ways, underscoring the diversity and significance of paradoxes as drivers of knowledge advancement. Building upon this exploration, Section §5 will focus on the black hole paradoxes' role in advancing our understanding of Quantum Gravity. Section §6 will conclude by outlining a possible heuristic approach to understanding and utilising paradoxes in modern scientific endeavours.

2. Which Paradoxes?

Paradoxes, often characterised as self-contradictory or counter-intuitive statements, have been extensively utilised in the fields of science, mathematics, and philosophy. Scholars have made significant efforts to provide a comprehensive definition of paradoxes and establish valuable classifications for their diverse manifestations. Sorensen's work in philosophy[2], for instance, offers an insightful analysis of paradoxes, presenting a proper definition and taxonomy that considers their roles, arguments, and structures. However, due to the multitude of paradoxes with varying characteristics, the task of providing a general definition remains challenging. For the sake of concreteness, I will consider for the rest of this paper the definition that Sainsbury gives[3]:

> This is what I understand by a paradox: an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises. Appearances have to deceive since the acceptable cannot lead by acceptable steps to the unacceptable. So, generally, we have a choice: either the conclusion is not really unacceptable, or else the starting point, or the reasoning, has some non-obvious flaw.

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2 - R. Sorensen, *A Brief History of the Paradox: Philosophy and the Labyrinths of the Mind,* 2003, Oxford University Press.

3 - R. Sainsbury, *Vagueness, ignorance, and margin for error.* The British Journal for the Philosophy of Science 46 (4), 1995, pp. 589-601.

According to this definition, paradoxes are significant as they require careful consideration. They serve as valuable tools in exposing false assumptions and challenging our initial beliefs by revealing that what may appear as unacceptable conclusions is merely counterintuitive. Paradoxes play a crucial role in pushing the boundaries of our understanding and encouraging deeper inquiry into complex phenomena. To comprehend the specific types of paradoxes relevant to physics and black hole paradoxes, this paper begins by examining various classifications of paradoxes based on their subjects, origins, or structure. Each classification will be evaluated for its relevance to the paradoxes of interest, shedding light on their significance in the context of our exploration. By considering different ways of categorising paradoxes, we aim to gain a deeper understanding of their nature and their specific relevance to the field of physics, particularly in studying black holes.

The first potential approach to classifying paradoxes is based on their subjects, i.e. according to the specific domains or topics they pertain to:

> *Ontological Paradoxes,* which pertain to theoretical models associated with real phenomena that give rise to a contradiction within the confines of the paradigm under discussion.
> *Semantic paradoxes*, which are those that deal with the explanation of real phenomena. They can be of theoretical (explanatory) character or observational character.

This first classification turns out to be tangential to the main topic of this paper since it revolves around one's stance regarding realism or antirealism concerning physical phenomena, i.e., in a nutshell, if one believes that unobservable entities postulated by modern physics have to be considered real or not. Specifically, within the context of black hole paradoxes, for instance, it relates to the attitude one holds toward the ontological status of black holes in Quantum Gravity. If one adopts a realist perspective and considers quantum black holes, i.e. the quantum gravitational objects that behave like general relativistic black holes at low enough energies, to be genuine entities, paradoxes associated with black holes can be categorised as ontological paradoxes. These paradoxes would then pertain to the inherent contradictions or challenges posed by the existence and nature of real black holes within the framework of Quantum Gravity. Conversely, if someone views quantum black holes as mere theoretical constructs or provisional descriptions that surpass the limitations of current theories, black hole paradoxes can be classified as semantic paradoxes. These paradoxes would primarily involve issues of language, meaning, and the interpretation of concepts associated with quantum black holes.

Another classification of paradoxes in terms of their subjects is proposed by Sainsbury:[4]

> *Scientific paradoxes* are created by a scientific understanding of the world in which observations, experiments and calculations play a crucial role.
> *Philosophical paradoxes* originated from a philosophical understanding of the world that cannot be profitably incorporated into science.

The paradoxes we are interested in can be classified as *scientific paradoxes*, as concrete scientific theories generate them. These paradoxes primarily arise from calculations within a specific, well-defined physical theory rather than being based on observations or experiments. Our focus will thus be on *scientific paradoxes*, aiming to provide additional elements that differentiate between various paradoxes in the realm of physics.

Another potential classification of paradoxes involves categorising them based on their *cause*:

> *Paradoxes of errors* caused by a mistake that can be difficult to recognise. If the error is discovered, the paradox is solved. Thus, the essential characteristic of this type of paradox is that to solve it means to improve the understanding of the theory. However, the paradox is not fatal: the solution to the paradox does not change the paradigm but helps to understand the theory under consideration deeply.
> *Paradoxes of contradictions are* caused by a contradiction between different principles in a physical theory. To solve the paradox, one of the instances should be dropped. Thus, the essential characteristic of this type of paradox is that solving it means drastically changing the physical theory under study. Thus, it is fatal: the solution to the paradox changes the paradigm.

Paradoxes of contradictions hold particular significance as potential guides for scientific inquiry. They are instrumental in critical junctures where transformative paradigm shifts occur in physics. Resolving paradoxes of error may identify a mistake but does not fundamentally alter our understanding of the theory in question. On the other hand, resolving paradoxes of contradiction entails substantial modifications to our existing physical principles, thereby reshaping our understanding of the subject matter. We will see in the following chapters that the paradoxes discussed in this paper, including black hole paradoxes, can be classified as paradoxes of contradictions. Specifically, all of these paradoxes stem from inconsistencies between well-established principles of our most advanced physical theories.

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4 - R. Sainsbury, *Paradoxes,* Cambridge University Press, 2009.

Scientific paradoxes can also be categorised based on their genesis, specifically whether they arise from observational or theoretical contexts in physics.:

Experimental paradoxes appear when experiments are performed, and the results of such experiments have no satisfactory explanation.
Theoretical paradoxes appear when a theoretical explanation of a physical phenomenon has not been precisely formulated.

In this regard, our primary focus lies on theoretical paradoxes, as one of the main motivations of this paper is to explore how to navigate the realm of science when our theories surpass experimental validation. Therefore, we envision a scenario where paradoxes arise not from experiments but instead from theoretical contradictions within a specific theory. Concerning theoretical paradoxes, an additional classification can be contemplated:

Paradoxes that are thought experiments, i.e. paradoxes created as a result of theoretical speculations.
Paradoxes that are not thought experiments, i.e. paradoxes created by explaining a physical phenomenon.

In addition to the classifications above, it is crucial to consider paradoxes that arise from thought experiments. While it is important to acknowledge potential experiments that may be feasible in the future, we should not limit ourselves to paradoxes solely concerning real physical phenomena, as empirical verification may not always be attainable. Paradoxes stemming from theoretical speculations hold the potential to provide valuable guidance for progress, much like paradoxes arising from the explanation of observable physical phenomena. By embracing theoretical paradoxes, we expand the scope of inquiry and open ourselves to new avenues of intellectual exploration.

After considering various classifications of paradoxes in philosophy and physics, I have identified key characteristics that make paradoxes useful for guiding progress beyond experimental boundaries. These characteristics include:

- Being physical paradoxes
- Having a theoretical nature
- Involving contradictions between widely accepted physical principles
- Potentially encompassing both real-world phenomena and thought experiments

In the next section, I will present examples of paradoxes in modern theoretical physics that possess these characteristics. By analysing these paradoxes' role in advancing scientific knowledge, we aim to understand the potential contribution of black hole paradoxes in contemporary research.

3. Paradoxes in Modern Theoretical Physics

This section will examine three examples of theoretical paradoxes arising from contradictions between widely accepted physical principles. These paradoxes served to illuminate the underlying structure of more fundamental physical theories. It is important to note that the content of this section should be viewed as a conceptual reconstruction of the paradoxes rather than a historical account. The primary objective is to identify the genuine contradictions that these paradoxes have revealed retrospectively.

The first paradox we will examine is the *train paradox* (§3.1), which highlights contradictions between Newtonian Mechanics and Electromagnetism. The second paradox is the *elevator paradox* (§3.2), which arises from the conflict between the Newtonian theory of gravitation and Einstein's foundational principles of Relativity. Lastly, we will discuss the *EPR paradox* (§3.3) concerning entanglement in Quantum Mechanics. In Section §4, we will explore the contributions of these three paradoxes to scientific progress, each in its own unique way.

1.1. The Train Paradox

The train paradox can be understood as the clash of three well-established physical principles:

(i) **The Principle of Relativity**: the laws of nature are the same in different frames of reference.

(i) **The Principle of the Constancy of the Speed of Light**: the speed of light in free space (vacuum) has the same value, c, in all inertial systems, i.e. the speed of light is a law of nature.

(i) **Galilean Relativity**: the laws of mechanics are the same for all inertial observers, i.e. for observers standing still or moving at constant velocity v.

Each of these principles seems entirely plausible on its own. However, as Einstein pointed out,[5] they can't all be true simultaneously. To demonstrate the inconsistency between these three principles, Einstein formulated a thought experiment that serves as the first paradox we will examine.

Imagine a scenario where we have two observers, Alice and Bob, as illustrated in Figure 1. Alice is situated on a train moving with a velocity of v relative to Bob, who is standing stationary on the ground. In this setup, let us assume that Alice possesses a light bulb, and light propagates throughout the train in accordance with Maxwell's equations, behaving as an electromagnetic wave with a velocity of c.

5 - A. Einstein, *Zur elektrodynamik bewegter körper,* Annalen der physik 4, 1905.

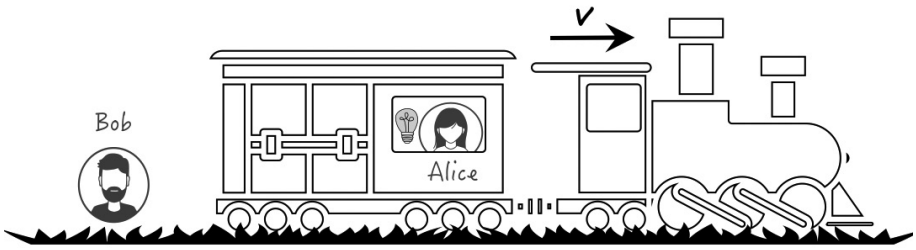


Figure 1: Illustration of the train paradox scenario. Observer Alice is on a moving train while observer Bob stands stationary on the ground. The train moves with velocity v, and Alice possesses a light bulb from which light propagates with velocity c.

If (i) is true, then according to the principle of relativity, the laws of nature should be the same for both Alice and Bob, regardless of their reference frames. However, based on (iii), Bob should observe light propagating with a velocity of , which contradicts

(ii). Hence, we encounter a paradox where the three well-established principles (i), (ii), and (iii) conflict with each other. It becomes necessary to abandon one of these principles to resolve this paradox.

Determining which principle to drop to resolve the paradox was challenging, as it often is with paradoxes of contradiction, including black hole paradoxes. Principle (i) appeared to be a fundamental and widely applicable requirement for any physical theory. On the other hand, dropping (ii) seemed more plausible because Newtonian mechanics' laws were invariant under Galilean transformations, satisfying (iii). Consequently, discarding (ii) would imply that Newtonian mechanics and the Newtonian theory of gravitation needed to be reexamined and reformulated. In contrast, it was already known that Maxwell's equations were not invariant under Galilean transformations. Additionally, Maxwell's equations revealed that the speed of light was determined by vacuum electromagnetic constants μ_0 and ε_0 and was independent of any preferred frame or observer. In other words, electrodynamics did not conflict with dropping (iii) and supported the validity of (ii). From this perspective, the paradox can be seen as a clash between Newtonian mechanics, which would entail dropping (ii) to solve the paradox, and Maxwell's electromagnetism, which would entail dropping (iii). A potential resolution to the paradox, aimed at preserving Newtonian Mechanics, involved dropping principle (ii) and suggesting that the speed of light, like any other entity, could vary depending on an observer's relative velocity. However, experiments designed to detect such varia-

tions in the speed of light, such as the Michelson-Morley experiment,[6] failed to detect any differences. Einstein proposed an alternative approach by dropping principle (iii) and exploring the consequences of this proposal. By abandoning (iii), it implied that Newtonian Mechanics and Gravity could not be entirely correct, resulting in the loss of gravity but gaining electromagnetism. This line of thinking ultimately led to the development of the theory of Special Relativity, which entailed a profound reevaluation of our entire understanding of space and time. Thus, the resolution of the paradox, by addressing the contradiction, guided scientific research towards a new paradigmatic perspective.

1. The Elevator Paradox

The Principle of Relativity, as discussed previously, asserts that the laws of physics should be consistent in different frames of reference. However, if we relinquish principle (iii) - Galilean Relativity - it becomes necessary to reformulate Newton's theory of gravitation, which accurately describes planetary orbits around the Sun and the gravitational effects of the Moon and the Sun on tides. This section aims to explore a second paradox that arises from the incompatibility between Newton's theory and Relativity and how Einstein, in his works,[7] managed to resolve this paradox. The focal point of our discussion will be the relationship between gravity and inertia. By resolving this paradox, we will quickly arrive at a geometric interpretation of gravity. First of all, let me recall that in Newtonian physics, there are two *a priori* different concepts of mass:

- The *inertial mass* m_{i} can be understood as the resistance of a particle against acceleration. It appears, in particular, in the second law of Newtonian Mechanics.
- The *gravitational mass* m_{g}, which can be understood as the "gravitational charge" of a particle.

As it is widely recognised, although gravitational and inertial mass are conceptually distinct physical quantities, they are empirically indistinguishable. In Newton's theory, the assumption that $\frac{m_i}{m_g}$ (where m_{i} represents the inertial mass and m_{g} represents the gravitational mass) would not lead to any immediate inconsistency. However, empirical evidence has consistently shown that the gravitational

6 - A. A. Michelson and E. W. Morley, *On the relative motion of the earth and of the luminiferous ether.* Sidereal Messenger, vol. 6, 1887, pp. 306-310.

7 - A. Einstein, *Die feldgleichungen der gravitation.* Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin 1915), pp. 844-847.

and inertial masses are indeed equal. This empirical equality caught Einstein's attention, leading him to believe that it was not a mere coincidence but rather a profound insight into the nature of gravity. It is important to note that this equality is a peculiar characteristic of gravity. In contrast, other fundamental forces, such as electromagnetism, do not exhibit this feature. For example, the equation describing the motion of an electrically charged particle in an electrostatic field does not exhibit an equivalence between the electric charge (analogous to inertial mass) and the force experienced (analogous to gravitational mass). The equality between the "charge" and the inertial mass is a very peculiar feature of gravity that is not shared by other forces.

Under the assumption that inertia and gravitation are distinct concepts despite their equal masses, we proceed to investigate their implications. By considering this hypothesis, we entertain the idea that these two phenomena represent separate physical entities, potentially leading to new understandings of their underlying principles. With this perspective in mind, we now turn our attention to the examination of two principles that arise from this hypothesis:

(i) **The Principle of Relativity**: the laws of nature are the same in different frames of reference.
(i) **Gravity Inertia**: Gravity and inertia are conceptually different.

To illustrate the conflict between these two principles, Einstein presented a paradoxical scenario known as the *elevator thought experiment* in Figure 2.

Consider an observer named Bob, who is situated inside a small elevator in outer space. In the first scenario (box A in Figure 2), the elevator is freely floating, and Bob experiences no forces acting on him. Now, imagine the elevator being subjected to a constant upward acceleration by an external force, as shown in box B of Figure 2. According to Newton's second law, Bob would be pressed against the elevator floor with constant force. Next, envision the same elevator being placed in a constant gravitational field and fixed to the roof by a resistant cable, as in box C of Figure 2. Again, Bob would feel a constant force pressing him against the floor. Since the gravitational and inertial masses are assumed to be equal, the force experienced by Bob in the accelerating frame (box B) and the force experienced due to the presence of the gravitational field (box C) should be equal. As a result, there is no experiment Bob can conduct to differentiate between the effects of a gravitational field and a constant external acceleration.

According to principle (i), the laws of nature governing the physics inside boxes B and C should be the same. However, if the principle (ii) holds, they should be different since the former is an effect of inertia, while the latter is an effect of

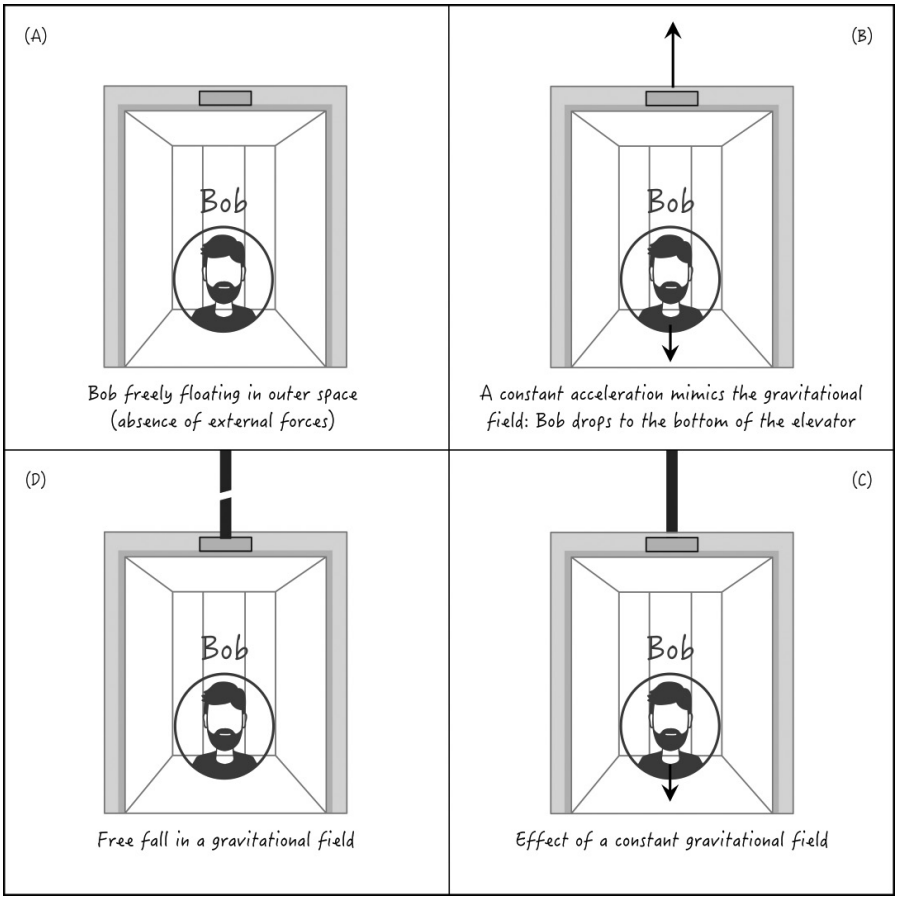


Figure 2: Illustration of the elevator thought experiment. Four scenarios are depicted: (A) Bob floating freely in the elevator in outer space, (B) the elevator undergoing constant (non-gravitational) acceleration, (C) the elevator in a constant gravitational field with a resistant cable, and (D) the cable cut, causing the elevator to fall freely. The paradox arises from the equivalence of the forces experienced by Bob in scenarios (B) and (C) despite the conceptual differences between inertial forces and gravitational forces.

gravitation. Now, consider the cable of the elevator being cut, as illustrated in box D of Figure 2. In this case, the elevator would freely fall downward, and Bob would float inside the elevator as if he were in outer space (box A of Figure 2). Thus, locally, the effects of gravity can be eliminated by transitioning to a freely falling reference frame. In other words, if the principle (i) is valid, the laws of na-

ture describing the physics inside boxes A and D should be the same. However, this contradicts principle (ii) since the absence of inertial forces in box A should conceptually be equal to the absence of gravitation, which is not the case. This paradoxical situation highlights the conflict between the principles of inertia and gravitation, leading to the need for a new theoretical framework to resolve the contradiction.

Facing this paradoxical situation, choosing between dropping one of the two principles became necessary. Einstein chose to drop principle (ii), considering gravity and inertia as the same phenomenon. This decision was not solely based on experimental evidence. It was also guided by the thought experiment, which highlighted the necessity of this choice for the principle of relativity to hold. It is said that Einstein referred to this as his "happiest thought." The insight led to the formulation of the Einstein Equivalence Principle, which states:[8]

> Gravitational and inertial effects are conceptually unified, representing the components of the same (possibly curved) compatible connection.

In other words, according to the equivalence principle, if you are in a small, enclosed space and unable to observe the outside world, you would be unable to distinguish between being in a gravitational field (such as on the surface of the Earth) and being in a uniformly accelerating frame of reference (such as an elevator accelerating upwards). Building upon the equivalence principle, Einstein developed the theory of General Relativity, which takes the laws of physics locally to be those of Special Relativity. General Relativity provides a framework for understanding the behaviour of objects and the curvature of spacetime in the presence of matter and energy. In this theory, gravity is not considered a force in the traditional sense but rather a consequence of the curvature of spacetime caused by the presence of matter or energy.

In this section, we have examined the conflict between Newtonian Gravitation and relativity, specifically regarding the relationship between gravity and inertia. While it is commonly believed that the conflict arises from Newtonian theory's instantaneous nature of gravitational forces, which contradicts the principle of no faster-than-light signals in Special Relativity, our analysis provides a different perspective. We have seen that the conflict between Newtonian Gravitation and relativity is not directly related to the speed of light limit imposed by Special Relativity. Instead, it arises when we consider that the conflict between Newtonian Gravitation and relativity stands from the fact that if we would like (i) to hold, we have to

8 - The precise statement of the Einstein Equivalence Principle has been adopted from J. Read and N. J. Teh. *Newtonian equivalence principles*. Erkenntnis 2022, pp. 1-25.

drop (ii), i.e., accept that gravity and inertia are the very same thing. This has nothing to do with special relativity and the bound on signals imposed by the speed of light limit. And indeed, one can also think of a non-relativistic gravitation theory compatible with (i): the Newton-Cartan theory.[9] The elevator paradox thought experiment led us to consider the possibility of locally eliminating gravity's effects. Einstein's solution, the Einstein equivalence principle, assumes that the laws of physics locally follow those of Special Relativity. However, this does not preclude the existence of alternative non-relativistic theories of gravity, such as the Newton-Cartan theory, which could be compatible with principle (i) by allowing for different laws of physics locally. Indeed, this choice would give rise to new non-relativistic theories of gravity compatible with (i).[10] As we would expect from this reasoning, it has been shown recently that these non-relativistic theories of gravity obey the Einstein equivalence principle. Of course, these theories would violate the speed of light limit, and they cannot be (most probably) the most fundamental theories we can build since they conflict with Special Relativity. However, they can be good gravitational theories, compatible with the principle of relativity, for describing low-velocity physics.

1. The EPR Paradox

The EPR paradox[11], named after Albert Einstein, Boris Podolsky, and Nathan Rosen, is a thought experiment in quantum mechanics that aims to show the incompleteness of Quantum Mechanics. The EPR paradox is based on "entanglement," a property of specific pairs of particles that are created together and remain connected, even when separated by large distances. In quantum mechanics, the properties of entangled particles are intertwined, and the state of one particle can instantaneously influence the state of the other, regardless of the spatial separation between them.

The EPR paradox emerges as the incompatibility of the conjunction of three principles:

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9 - E. Cartan, *Sur les variétés à connexion affine et la théorie de la relativité generalisée (première partie)*, in «Annales scientifiques de l'École Normale Supérieure» 1923, Volume 40, pp. 325-412. For a modern take on Newton-Cartan theory and its generalisations see M. H. Christensen, J. Hartong, N. A. Obers, and B. Rollier, *Torsional newton-cartan geometry and lifshitz holography*. Phys. Rev. D 89 2014.

10 - J .Hartong and N. A. Obers, *Hořava-Lifshitz gravity from dynamical Newton-Cartan geometr* , in «Journal of High Energy Physics» 2015 (7), pp. 1-44.

11 - A. Einstein, B. Podolsky, and N. Rosen. *Can quantum-mechanical description of physical reality be considered complete?*, Phys. Rev. 47 1935, pp. 777-780.

(i) **Realism**: if, without interacting with a given system, it is possible to predict with certainty (i.e. with probability one) the value of a particular physical quantity, this physical quantity corresponds to an objective property of the system, i.e. a property independent of any observers.
(i) **Locality**: given two physical systems and supposing that during a specific time interval, they remain isolated from each other (which is achieved in Special Relativity if they are space-like related), then the evolution of the physical properties of one of them during this time interval cannot be influenced by operations performed on the other.
(i) **Completeness**: if any element of the objective reality has a counterpart among the concepts used by a given theory, then the theory is said to be complete.

The landmark paper by Einstein, Podolsky, and Rosen presented a thought experiment known as the EPR paradox, which revealed a fundamental conflict between well-established principles in quantum mechanics. This paradox raised the question of whether we need to abandon one of these principles to resolve the contradiction. This section will examine the Bohm version of the EPR paradox.

Consider a source that emits a large number of pairs of spin-1/2 particles. These particles are entangled and are in a special state known as the singlet state. The singlet state is a maximally entangled state where the total spin of the particle pair is zero. The singlet state can be represented as:

$$|\psi >= \frac{|\uparrow\downarrow> - |\uparrow\downarrow>}{\sqrt{2}}$$

where $|\uparrow>$ and $|\downarrow>$ represent the spin-up and spin-down states, respectively, and the subscripts 1 and 2 denote the two particles of the entangled pair. It is important to note that the singlet state is such that the measurement of the spin of one particle instantaneously determines the spin of the other particle, regardless of the distance between them. This property intrigued Einstein, Podolsky, and Rosen and led to the formulation of the EPR paradox. Consider a scenario where two experimenters, Alice and Bob, collect pairs of entangled electrons emitted by the bulb, as in Figure 3.

At an initial time t_0, the spatial wave functions of the electrons are separated by a distance d such that they cannot interact without violating the locality principle. At a later time $t_1 > t_0$, Alice decides to measure the spin of her electron along the z-axis. According to the principles of quantum mechanics, when Alice performs the measurement, the state of her electron collapses to either spin up or spin down with equal probability. Let us assume Alice measures a spin-up result, causing the

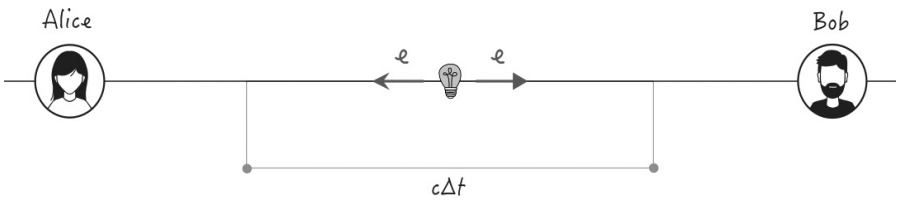


Figure 3: The EPR thought experiment illustrating the measurement of entangled electron spins. At an initial time t_0, the entangled electrons are separated by a distance d and cannot interact. At a later time t_1, Alice measures the spin of her electron along the z-axis, which causes the overall system state to collapse. The paradox arises from Bob's electron spin's instantaneous determination, violating the locality principle.

overall system state to collapse to the entangled state $|\uparrow\downarrow>$. If we assume the validity of the principle of locality (ii), which states that measurements performed by Alice cannot have an instantaneous effect on Bob's electron, then Bob's electron should have had a definite spin even at the initial time t_0.[12] This implies that before t_0, the pair of electrons was in the singlet state, and after t_0, the state of the electrons became $|\uparrow\downarrow>$. The experiment can be repeated multiple times on each pair of electrons, resulting in a collapse of the state to either $|\uparrow\downarrow>$ or $|\downarrow\uparrow>$ with equal probability. As a result, the state between the initial time t_0 and the measurement time t_1 becomes a mixed state given by:

$$\rho = \frac{|\uparrow\downarrow><\uparrow\downarrow|}{2} + \frac{|\uparrow\downarrow><\uparrow\downarrow|}{2}$$

This introduces an inconsistency: since no measurements have been performed before t_1, the unitary evolution of the system should be preserved. However, the transition from the initial pure state (4) to the mixed state (5) violates unitarity. Furthermore, the different unitary evolutions associated with the initial pure state and the subsequent mixed state lead to distinct predictions for the mean values of various operators, such as spin x or y.

We can now observe the conflict between the three principles:

- The conjunction of (i) and (ii) leads to a violation of a fundamental principle of quantum mechanics. Thus (iii) cannot hold, i.e. quantum mechanics is not a complete theory.

12 - Module Separability, which is implicitly assumed in Locality by Einstein and collaborators.

- (iii) holds, i.e. quantum mechanics is complete. Thus either (i) or (ii) cannot hold. Otherwise, we fall into a contradiction.

The paradox could be thus resolved by dropping one of the three principles:

Drop (i): One possible resolution is to abandon Realism, suggesting that the value obtained from a measurement cannot be attributed to an objective physical property independent of the measurement process. Even if we have knowledge of the system's dynamics, we cannot extrapolate the measurement result back to the time before the measurement took place. Consequently, there is no contradiction for the time interval $t_0 < t < t_1$.

Drop (ii): Another option is to reject Locality by proposing that a measurement can instantaneously affect the wave function at arbitrarily large distances. Consequently, information cannot be extrapolated backwards in time, even for systems that are separated by space-like intervals. This viewpoint contradicts the principles of Special Relativity. However, there are ways to render entanglement and Special Relativity compatible, even considering this second option. Indeed, it should be noted that when gravitational interactions are taken into account, a different interpretation of Locality becomes necessary.

Drop (iii): Another possibility is to reject Causality by suggesting that the spins of Alice and Bob were already predetermined before Alice's measurement, but their values were not accessible through the singlet state (4). This implies that the singlet state provides only probabilistic information as an average over "hidden" variables, representing the true underlying variables. An analogy can be drawn with temperature in Statistical Mechanics, which is an average over a large number of particles whose individual states are not measured. In this view, the singlet state does not capture the complete information about the system, and additional hidden variables exist that determine the actual values of the spins.

Option three of this list has been ruled out by Bell[13], who demonstrated the incompatibility between quantum mechanics and local hidden variable theories. Consequently, it is not feasible to drop (iii) by proposing alternative theories. This implies that quantum mechanics stands as a complete theory. Therefore, the only possible resolutions to the EPR paradox are those that abandon either Realism or Locality.

13 - J. S. Bell and J. S. Bell, *Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy*, Cambridge University Press, Cambridge 2004.

4. On the Role of Paradoxes for Scientific Progress

In the preceding section, we examined three well-known paradoxes from the past century and their significant contributions to the development of physical theories, particularly Relativity and Quantum Mechanics. These paradoxes, derived from thought experiments, present physical contradictions that have proven instrumental in advancing our understanding of the natural world. In this section, we will analyse the specific role played by these paradoxes in driving scientific progress. Furthermore, we will explore the lessons we can glean from these paradoxes regarding the significance of paradoxes in contemporary research that may lack experimental validation. The first paradox, as discussed in Section 1.2.1, directly impacted the construction of theories. It can be said that Einstein's Special Relativity theory is rooted in the train paradox. As stated in Einstein's own words in "Relativity: The Special and General Theory"[14]:

> If a ray of light is sent along the embankment, we see from the above that the tip of the ray will be transmitted with the velocity c relative to the embankment. Let us suppose that our railway carriage is again travelling along the railway lines with the velocity v and that its direction is the same as that of the ray of light, but its velocity is much less. Let us inquire about the velocity of propagation of the ray of light relative to the carriage. We can here apply the consideration of the previous section since the ray of light plays the part of the man walking along relatively to the carriage. The velocity W of the man relative to the embankment is replaced by the velocity of light relative to the embankment. w is the required velocity of light with respect to the carriage, and we have
>
> $$W = c - v\,.$$
>
> The propagation velocity of a ray of light relative to the carriage thus comes out smaller than c.

Einstein's presentation of the train paradox highlights its crucial role in shaping the development of Special Relativity. Confronted with the inconsistency between the three principles of Relativity, Constancy of the Speed of Light, and Galilean Relativity, Einstein decided to abandon the latter principle. This choice led him to seek a new form of transformation that would maintain consistency with Relativity and the Constancy of the Speed of Light. By resolving this paradox, Einstein laid the foundation for formally deriving the theory of Special Relativity, which has revolutionised our understanding of space and time.

14 - A. Einstein, *Relativity*, Routledge, London 2013.

The second paradox we examined, the elevator paradox, offers a different perspective on scientific progress. Unlike the train paradox, which directly contributed to the development of Special Relativity, the elevator paradox did not play a direct role in formulating General Relativity. Einstein proposed the elevator paradox later, drawing inspiration from the experiment involving two softballs in an otherwise empty universe, one spinning and becoming ovoid while the other remained unchanged. However, the elevator paradox still holds significance in scientific progress. By reconstructing the paradox and exploring its implications, we were able to identify two distinct inconsistencies: the conflict between the Galilean nature of Newton's Gravitation and the Constancy of the Speed of Light and the inconsistency between the Principle of Relativity and the conceptual separation of gravity and inertia. Through this analysis, we determined that the elevator paradox, and subsequently the foundation of General Relativity, primarily addresses the latter inconsistency.

Indeed, as mentioned at the end of section §3.2, Einstein's solution to the elevator paradox, known as the Einstein equivalence principle, is also shared by covariant formulations of non-relativistic theories of gravity. This observation suggests that the elevator paradox not only played a role in the development of General Relativity but also motivated the exploration of new theories of gravitation with a non-relativistic character. By carefully analysing the elevator paradox, researchers could generalise General Relativity and discover a broader family of gravitational theories that differ in the local group of spacetime considered. Although these new theories can- not be considered fundamental descriptions of our world due to their conflict with the constancy of the speed of light (as they are either non-relativistic or ultra-relativistic), they serve as coarse-grained descriptions that can accurately describe physical phenomena at low velocities. In fact, these theories have proven to be fundamental in various physical situations.[15]

Interestingly, it has been demonstrated that these new non-relativistic theories of gravitation pass the classical tests of General Relativity, such as the anomalous precession of the perihelion of Mercury, the bending of light in gravitational fields, and the gravitational redshift.[16] This suggests that the classical tests are not explicitly testing General Relativity itself but rather the broader family of gravitational theories that share the common feature of the Equivalence Principle. James Read and collaborators argue that the classical tests of General Relativity are test-

15 - S. Mukohyama, *Hořava-Lifshitz cosmology: a review*, in «Classical and Quantum Gravity» 27 (22), 2010. D. T. Son, *Newton-Cartan Geometry and the Quantum Hall Effect*.

16 - D. J. Hansen, J. Hartong, and N. A. Obers. *Gravity between Newton and Einstein*, in «International Journal of Modern Physics» D 28 (14), 2019.

ing the Equivalence Principle itself.[17] The presence of gravitational waves emerges as a distinguishing feature between General Relativity and the non-relativistic theories of gravitation. Therefore, the detection of gravitational waves, achieved approximately one hundred years after the formulation of General Relativity,[18] represents the first real test of its validity. In summary, the elevator paradox has not only played a crucial role in the development of General Relativity but has also provided insights into the empirical status of General Relativity and alternative theories of gravitation. By examining this paradox more closely, researchers have better understood the ontology of gravitational theories in general.

The EPR paradox, unlike the train and elevator paradoxes, did not directly lead to the development of a new theory to resolve the inconsistency. Instead, it played a significant role in deepening our understanding of the existing theory, Quantum Mechanics. The EPR paradox allowed us to explore Quantum Mechanics more profoundly and gain insights into its interpretive and metaphysical aspects. Following the empirical tests of Bell's inequality, we have strong evidence to support the view that Quantum Mechanics is a complete theory of the microscopic world within the non-relativistic and non-gravitational domains. However, our understanding of what Quantum Mechanics implies about the nature of reality is still an active area of research. The philosophy of Quantum Mechanics has emerged as a vibrant field of study, driven in part by the pro- found questions raised by the EPR paradox. The EPR paradox has served as a crucial tool in examining various aspects of Quantum Mechanics, both at the interpretive and metaphysical levels. It has helped us better comprehend the fundamental features of quantum theory and shed light on the nature of reality in the quantum realm. While the EPR paradox did not contribute directly to the construction of new theories, it has played a pivotal role in advancing our understanding and highlighting the critical aspects of Quantum Mechanics.

We have observed that throughout the history of modern theoretical physics, paradoxes have served three distinct roles:

> *Construction of a theory*: Paradoxes have often played a crucial role in constructing more fundamental theories. By revealing inconsistencies or contradictions within existing theories, paradoxes have motivated the search for new frameworks that can resolve these issues. The train paradox, for example, led to

17 - James Read, Marco Sanchioni, W. Wolf, *Underdetermination in general relativity,* Manuscript 202x.

18 - Abbott, B. P., R. Abbott, T. Abbott, M. Abernathy, F. Acernese, K. Ackley, C. Adams,T. Adams, P. Addesso, R. Adhikari, et al, *Observation of gravitational waves from a binary black hole merger*, in «Physical review letters» 116 (6), 2016.

the development of Special Relativity as a more comprehensive theory that incorporates the principles of relativity and the constancy of the speed of light.

Development of a theory: Paradoxes have also contributed to the development of existing theories. They have acted as intellectual puzzles that challenge our understanding of a theory and have led to further investigations, refinements, and conceptual breakthroughs. The elevator paradox, for instance, played a significant role in formulating generic theories of gravitation by highlighting the inconsistency between the Galilean character of Newtonian gravitation and the principles of relativity and constancy of the speed of light.

Understanding of a theory: Paradoxes have served as valuable tools for deepening our understanding of physical theories' consequences and implications. They have prompted philosophical and metaphysical considerations, allowing us to delve into a theory's profound implications and interpretive aspects. The EPR paradox, for example, has been instrumental in exploring the philosophical foundations of Quantum Mechanics and shedding light on the nature of reality at the quantum level.

In the next section, we will explore how similar roles can be played by black hole paradoxes in contemporary physics. These paradoxes, like their predecessors, have the potential to contribute to the construction, development, and understanding of our current physical theories.

5. Paradoxes and Quantum Gravity

Over the past 50 years, the development of a coherent theory of quantum gravity has been a central focus in theoretical physics. This theory aims to unify General Relativity, which describes gravity as the curvature of spacetime, with quantum mechanics, which describes the behaviour of matter at the microscopic level. Black holes, with their fascinating properties, have played a crucial role in advancing our understanding and exploring new ideas in the quest for a theory of quantum gravity. Black holes provide a unique system where relativistic and quantum effects must be considered simultaneously. General Relativity is essential for describing black holes since they are objects characterised by intense gravitational forces that result in the curvature of spacetime, preventing even light from escaping. However, when we delve into the interior of a black hole, we encounter a singularity—a point in spacetime where curvature becomes infinite, and General Relativity breaks down. This singularity and its immediate surroundings constitute a microscopic region, suggesting the importance of quantum theory in its description. The combination of General Relativity and quantum theory is necessary to understand black holes fully. Therefore, gaining a deeper understanding of black

hole structures, particularly their quantum properties, is crucial for developing a theory of quantum gravity. Black holes serve as a testing ground for exploring the interplay between gravity and quantum mechanics, guiding researchers towards a unified framework encompassing both theories. By studying black hole paradoxes and unravelling their quantum behaviour, scientists hope to shed light on the nature of spacetime, the information paradox, the behaviour of matter under extreme conditions, and the fundamental principles underpinning our universe's fabric. These investigations are of utmost importance for contemporary theoretical physics, as they offer valuable insights into the nature of quantum gravity and the fundamental structure of our universe.

Since Stephen Hawking's groundbreaking paper on Hawking radiation in 1975, it has been widely recognised that attempting to describe the quantum properties of black holes by combining quantum mechanics and general relativity leads to inherent inconsistencies, giving rise to various paradoxes. These black hole paradoxes are of great interest as they hold the potential to provide insights into the realm of quantum gravity. Resolving these paradoxes is crucial as they have profound implications for fundamental questions related to the foundations of general relativity and quantum mechanics. One of the critical paradoxes is the information loss paradox, which directly impacts the question of whether quantum gravity is a unitary theory. According to this paradox, the process of black hole evaporation through Hawking radiation appears to destroy information, violating the principles of unitarity in quantum mechanics. Resolving this paradox is essential for reconciling quantum mechanics and general relativity and understanding the fate of information in black hole physics. Another significant paradox is the Page paradox, which concerns the nature of a black hole's microscopic degrees of freedom. It raises the question of whether the thermodynamic properties of a black hole, such as its entropy, can be derived from the statistical mechanics of its microscopic constituents, as is the case for other thermodynamic quantities. Resolving this paradox requires a deeper understanding of black holes' underlying microscopic structure and statistical behaviour. In recent years, the firewall paradox has gained considerable attention in the theoretical physics community. This paradox suggests the possible non-existence of the interior of a black hole and proposes the presence of a firewall – a region of extreme energy near the event horizon. This idea challenges our traditional understanding of black hole interiors and raises profound questions about the nature of spacetime and the fundamental principles governing black hole physics. Resolving these black hole paradoxes is crucial for advancing our understanding of quantum gravity and unravelling the mysteries of black holes. It involves delving into the nature of information, the microscopic constituents of black holes, and the fundamental behaviour of spacetime under ex-

treme conditions. By addressing these paradoxes, we can hope to unveil deeper insights into the nature of the universe and the unification of quantum mechanics and general relativity.

Analysing the conceptual foundations of black hole paradoxes and their resolutions can be approached through the three paradigms we discussed earlier. Let us explore each paradigm in the context of black hole paradoxes:

- *Construction of a theory*: By examining the logical resolution of these paradoxes, researchers can gain valuable insights into constructing a theory of quantum gravity. Understanding the inconsistencies and finding ways to reconcile quantum mechanics and general relativity in the context of black holes can provide crucial clues for formulating a more fundamental theory. This involves exploring new mathematical frameworks, incorporating quantum effects into the description of black holes, and addressing issues such as information preservation and the fate of evaporated particles.
- *Development of a theory*: Deeply analysing the conceptual structure of black hole paradoxes helps identify implicit assumptions and fundamental principles at play. By disentangling these principles and exploring alternative resolutions, physicists can develop new theoretical frameworks that extend our current understanding. This may involve revisiting the notions of locality, causality, and information preservation in the context of black hole physics. The resolution of paradoxes can lead to the development of alternative theories, modified gravity models, or novel approaches to quantum gravity.
- *Understanding of a theory*: Philosophical analyses of black hole paradoxes can offer insights into the nature of quantum black holes and the underlying ontology of these objects. By critically examining the paradoxes, researchers can uncover our current theories' fundamental assumptions and implications. This deep understanding of the paradoxes and their resolutions can inform philosophical and metaphysical considerations related to the nature of spacetime, information, and the emergence of classical behaviour from quantum systems. It allows us to explore the conceptual boundaries of our theories and gain a deeper understanding of the nature of black holes and the quantum universe.

By employing these three paradigms, researchers can approach black hole paradoxes from different angles, leading to advancements in theory construction, theory development, and a more profound understanding of the physical and philosophical implications of these paradoxes. This multidimensional approach is crucial for progress in the quest for a theory of quantum gravity and sheds light on the fundamental nature of black holes in the quantum realm.

Paradoxes are not to be feared but embraced as the most compelling and fertile grounds for exploration. While experiments remain the ultimate arbiter of truth in science, paradoxes offer theorists a valuable starting point when empirical data is lacking. They serve as catalysts for new theories, paving the way for unforeseen and exhilarating breakthroughs.

6. Conclusions

In this paper, we examined three significant paradoxes that have emerged in the past century and explored their crucial role in developing our most successful physical theories, namely Relativity and Quantum Mechanics. These paradoxes have driven scientific progress and shaped our understanding of the natural world. Paradoxes serve as catalysts for scientific inquiry and innovation by challenging existing frameworks and principles. They expose gaps or limitations in our current understanding and push scientists to explore alternative explanations and theories. The resolution of paradoxes often leads to paradigm shifts, where new concepts and principles are introduced to reconcile conflicting ideas.

We have seen how the analysis of paradoxes in theoretical physics reveals their multi-faceted roles in the development of scientific knowledge. Paradoxes serve as catalysts for theory construction, providing insights that lead to the formulation of more fundamental theories. They also contribute to the development of existing theories by unravelling inconsistencies and prompting further investigations into their conceptual foundations. Additionally, paradoxes deepen our understanding of physical theories, shedding light on their implications and opening avenues for philosophical and metaphysical considerations.

Through the examination of the train paradox, the elevator paradox and the EPR paradox, we have witnessed how paradoxes can motivate the birth of new theories, clarify the consequences of existing theories, and offer insights into the ontology of physical phenomena. Similarly, in the study of black hole paradoxes, we have seen how they present challenges and opportunities in the quest for a theory of quantum gravity. These paradoxes, such as the information loss paradox, the Page paradox, and the firewall paradox, confront us with fundamental questions that impact our understanding of the nature of space, time, and matter.

While experiments remain the ultimate arbiter of truth in science, paradoxes provide a fertile ground for theoretical exploration when empirical data is scarce. They ignite the intellectual curiosity of theorists and stimulate the development of alternative theories and conceptual frameworks. Paradoxes push the boundaries of our knowledge, inspiring new approaches and unexpected advances. In this light, paradoxes should not be feared or dismissed but embraced as essential components of scientific progress. They challenge our assumptions, drive innovation, and

expand the frontiers of knowledge. Paradoxes serve as powerful tools that complement experimental investigations and contribute to the richness and diversity of theoretical physics.

As we continue to unravel the mysteries of the universe, it is essential to appreciate the role of paradoxes in shaping our understanding of the laws of nature. Embracing paradoxes as opportunities for growth and discovery, we can navigate the complexities of theoretical physics and embark on a journey towards deeper insights and profound breakthroughs. In the ever-evolving landscape of scientific inquiry, paradoxes will continue to captivate the minds of researchers, pushing the boundaries of human knowledge and inspiring future generations of scientists to unravel the enigmas of the universe.

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